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Effects of Scanning Interval on Peak Load Measurements

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During periods of peak traffic in a communication system, it is well known that load measurements obtained by switch counts are biased upwards as a result of both variation in the source load and of the scanning process itself. This paper investigates the bias introduced into the expected value of a peak load measurement as a result of the scanning process. For small values of source load, we show that this effect is most pronounced when the scanning interval greatly exceeds the average holding time of the item being measured.

I. INTRODUCTION

The determination of the load carried by a group of circuits is of prime importance in facilities engineering. The present technique for acquiring data is the switch count method. Here the equipment is scanned at fixed intervals, and the total number of busy circuits is recorded. Dividing this total by the number of scans during the observation interval yields an estimate of the carried load in erlangs. Typically, a day may contain N such observation intervals. The daily bouncing busy-hour measurement is obtained by retaining the largest usage measurement of the N daily observation intervals. Averaging this quantity over a number of days should give an indication of average peak loads to be expected. Since there is some inherent error between the carried load of an observation interval and that determined by the switch count method, it is important to know the effect of this discrepancy on the expected bouncing busy-hour measurement.

II. SCANNING ERRORS AND BOUNCING BUSY-HOUR MEASUREMENTS

To quantitatively study this phenomenon, the following model for the process was chosen. Calls are assumed to arrive according to a Poisson process, holding times are exponentially distributed, and the

number of circuits is unlimited so that there is no blocking of incoming calls. All hourly source loads are equal, and this load is denoted by A. Each day is divided into N one-hour observation periods with the measurements made during individual hours statistically independent. (This is justified by the relatively short holding time compared to the one-hour observation interval.) Letting A_i denote the carried load of the ith hour, M_i the measured usage (obtained by scanning), and ϵ_i the discrepancy between the carried and switch count usage for the ith hour it is clear that

$$A_i = M_i + \epsilon_i \qquad i = 1, 2, \cdots, N. \tag{1}$$

Riordan[1] has shown that the load carried during any one observation interval is a random variable and fluctuates about the value of the source load. Letting $E(\ )$ denote expected value and $\sigma^2(\ )$ denote variance he has shown that

$$E(A_i) = A \qquad i = 1, 2, \cdots, N \tag{2}$$

$$\sigma^2(A_i) = \frac{2A}{\gamma^2}(\gamma - 1 + e^{-\gamma}), \tag{3}$$

where

γ = the ratio of the length of the observation interval to the average holding time.

Hayward[2] has shown that the error introduced by the switch count process is random with

$$E(\epsilon_i) = 0 \qquad i = 1, 2, \cdots, N \tag{4}$$

$$\sigma^2(\epsilon_i) \doteq \left(\beta \frac{1 + e^{-\beta}}{1 - e^{-\beta}} - 2\right) / (A\gamma), \tag{5}$$

where

β = ratio of scanning interval to mean holding time.
γ = ratio of the length of the observation interval to the average holding time.

When making load measurements, one has to contend with both the fluctuation of the carried load about the source load and the fluctuation of the measured (switch count) load about the carried load. It should be noted that Messerli[3] has shown that these two effects are uncorrelated, i.e.,

$$E(A_i\epsilon_i) = 0 \qquad i = 1, 2, \cdots, N, \tag{6}$$

which implies that

$$\sigma^2(M_i) = \sigma^2(A_i) + \sigma^2(\epsilon_i) \qquad i = 1, 2, \cdots N. \tag{7}$$

When making peak measurements, the mean bouncing busy-hour load

is given by

$$E(\text{BBH})_{\text{True}} = E\left(\begin{matrix} \max\{A_i\} \\ i = 1, 2, \cdots, N \end{matrix}\right), \tag{8}$$

while the mean, measured, bouncing busy-hour load is

$$E(\text{BBH})_{\text{Scanned}} = E\left(\begin{matrix} \max\{M_i\} \\ i = 1, 2, \cdots, N \end{matrix}\right). \tag{9}$$

It is the discrepancy between the quantities of (8) and (9) which this paper investigates. From Hayward's analysis, it is seen that the variance of the switch count error increases as the scanning interval increases. This implies that the variance of the switch count measurement also increases with increasing scanning interval. With more fluctuation of the measurement about its average (source load), one may expect a larger daily peak (from scanning) and, hence, an increase in the mean bouncing busy-hour measured usage. For a constant source load, it appears that the scanned bouncing busy-hour average will increase as the time between successive scans increases.

An expression was developed and used to compute the expected bouncing busy-hour measurement as a function of source load, mean holding time, scanning interval, and the number of hours of measurement per day. The probability generating function for the hourly measurement is computed and used to obtain the probability distribution for the measurement of one observation interval. With this, the distribution for the peak measurement of several observation periods is computed. The expected peak measurement is then easily calculated (see appendix). Figure 1 shows the expected bouncing busy-hour measurement as a function of the source load for a day with 8 hourly measurements where the mean holding time is 10 seconds. Both the 100-s scan and 10-s scan results are displayed here. In those instances where the source load is not constant during the entire day, the number of hours which contribute to the bouncing busy-hour measurement will be less than 8. Assuming that there are 2 hours in the morning which significantly contribute to the morning peak and 2 hours in the afternoon which account for the afternoon peak, the measurement is dependent only upon these 4 hours. Results for a 4-hour day are also shown in Fig. 1. As anticipated, these measurements are found to increase as the time between scans increases. The dependence upon scanning interval is more clearly shown in Fig. 2 where the source load is held constant and only the scanning interval is varied. With the 4-hour day, it is seen that the daily bouncing busy-hour measurement obtained by the 100-s scan is inflated 8.9 percent above the 5-s scan measurement. This discrepancy increases to 12.1 percent for an 8-hour day. A second point to be noted is that the measurement does not converge to the average load of 1 erlang as the time between

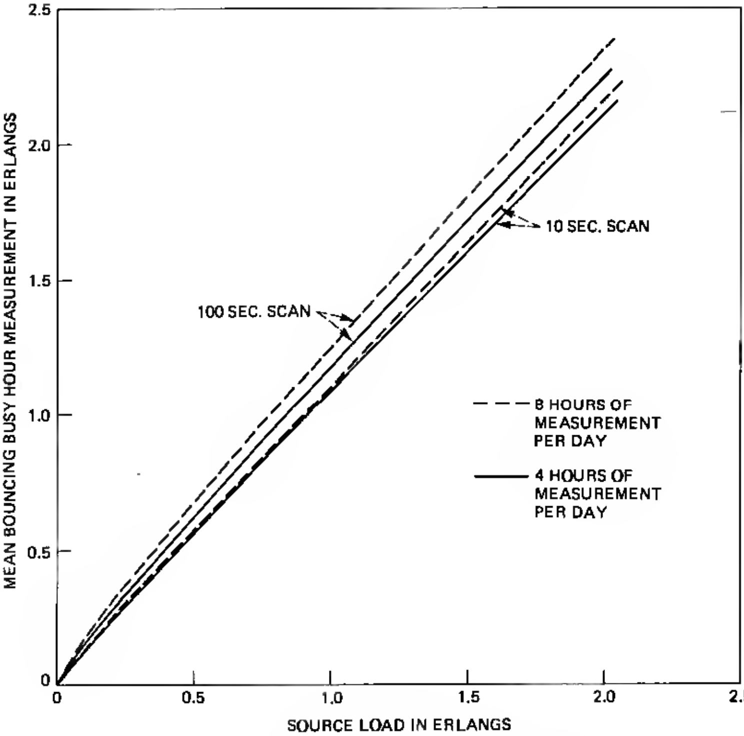

Fig. 1—Daily bouncing busy-hour measurements (mean holding time is 10 seconds).

scans approaches zero, but converges to $E(\text{BBH})_{\text{True}}$ [eq. (8)], which is strictly larger than 1 erlang. Even with no scanning bias, the average peak measurement (average peak carried load in the absence of scanning error) will exceed the numerical value of the source load.* This is an important difference between peak and average load measurements.

When the weekly peak hour of usage is retained and averaged, the effects of scanning error are even more pronounced than with daily measurements. Figure 2 also shows the dependence of the expected weekly peak hour measurement upon the scanning interval for a 40-hour week (5 days with 8 hours of measurement per day) and a 20-hour week (5 days with 4 hours of measurement per day). With the 20-hour week, the 100-s scan average is inflated 15.6 percent above the 5-second average, and for the 40-hour week this inflation becomes 17.9 percent. Figure 3 shows the expected weekly peak hour measurement

* As previously noted in Ref. 1, the carried load experienced on an infinite trunk group during one observation interval is a random variable whose mean equals the source load. Therefore, the expected value of the largest of several such random variables must exceed the expected value of any one hourly carried load, i.e., the source load.

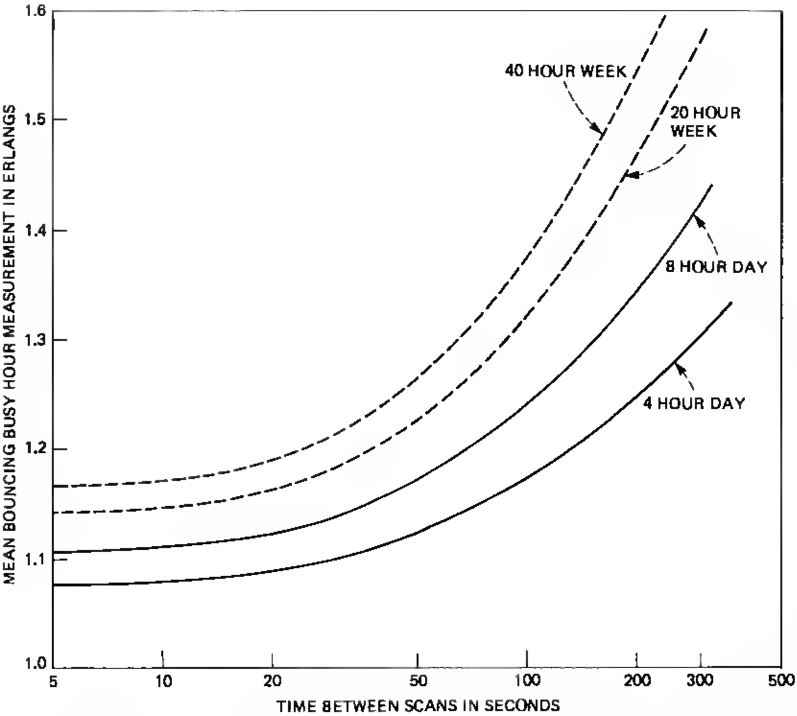

Fig. 2—Bouncing busy-hour measurements (10 seconds mean holding time, 1 erlang of source load).

as a function of source load. Here the average holding time is taken to be 10 seconds. Again it is observed that the expected peak measurement increases with both the time between scans and the number of observation intervals contributing to the peak.

Figure 4 shows the dependence of the daily bouncing busy-hour measurement upon the average holding time. The load is fixed at 1 erlang, and results are plotted for a 4- and 8-hour day with both the 100- and 10-s scan. With holding times larger than 200 s, there is excellent agreement between the 10-s and 100-s scan measurements. At these holding times, there is very little error introduced as a result of the scanning process, and consequently the 10-s and 100-s scan results would be expected to agree. However, at smaller holding times, the 100-s scan introduces a significant amount of error into the measurement. At this point, the graphs for the 100-s scan and the 10-s scan begin to diverge, and this discrepancy is seen to increase with decreasing holding time. Similar results are shown in Fig. 5 for weekly peak measurements.

The limiting value of these averages as the holding time approaches

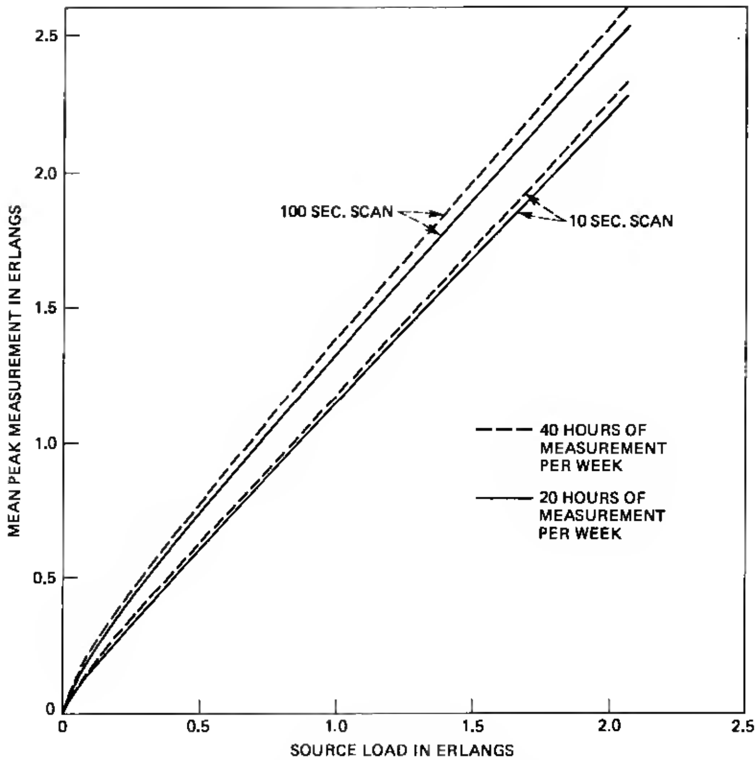

Fig. 3—Weekly peak hour measurements (mean holding time is 10 seconds).

zero (with a constant source load) is easily calculated. For short duration calls with a large number of holding times between successive scans, the measurements made at the scanning epochs are statistically independent. The distribution of the measurement at any one scanning epoch reduces to the state probabilities for the number of busy trunks (the Poisson distribution for this analysis). The hourly measurement (in switch counts) then becomes the sum of c (number of scans per hour) Poisson random variables, which is again Poisson. The distribution for the peak measurement then becomes the distribution of the maximum of several Poisson variables. Thus,

$$\lim_{\tau \to 0} E(\text{BBH})_{\text{Scanned}} = \frac{1}{c} \sum_{K=1}^{\infty} \left\{ 1 - \left(\sum_{j=0}^{K} \frac{(ca)^j}{j} e^{-ca} \right)^N \right\}, \qquad (10)$$

where

a = source of load

c = number of scans per hour

N = number of hours from which the peak is chosen.

$\tau =$ average holding time.

III. CONCLUSION

From the preceding analysis, it is clear that peak load measurements may be artificially inflated as a result of the scanning process. This is especially true in situations of small source loads when the average holding time is much shorter than the scanning interval. When using these load measurements with extreme-value engineering techniques, care must be taken to avoid over-engineering facilities. Once an engineering criterion is chosen, the results of this analysis may be used to estimate the magnitude of any over-engineering.

Two factors primarily affect peak value averages, one the number of hours contributing to the peak and the other the bias introduced by the scanning process. The peak measurement will increase with the number of observation intervals which contribute to the peak. In fact, it can easily be shown that, for the model used in this analysis, the expected peak measurement diverges as the number of observation intervals increases. With respect to the scanning bias, when choosing

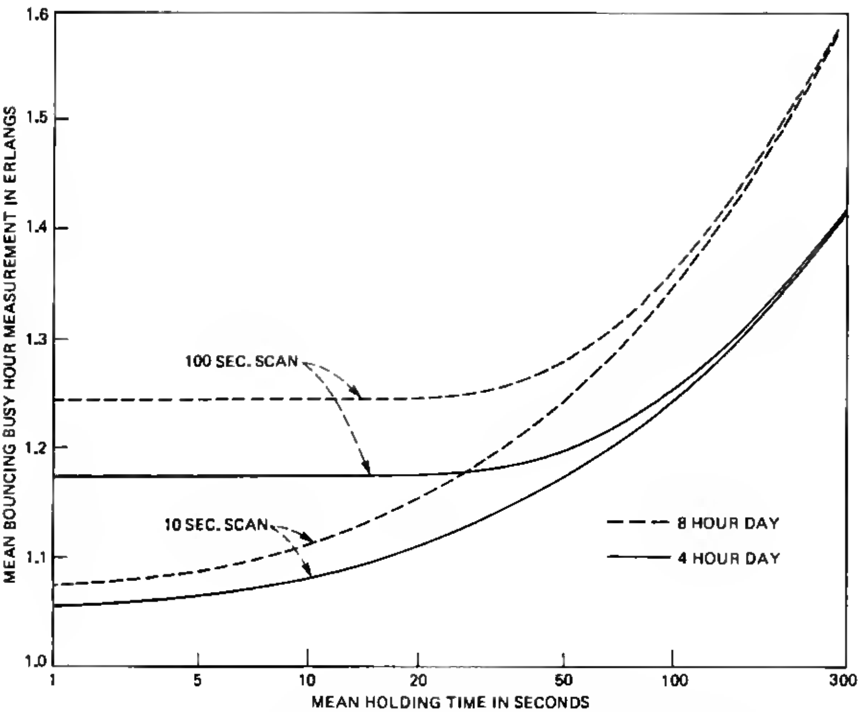

Fig. 4—Daily bouncing busy-hour measurements (1 erlang of source load).

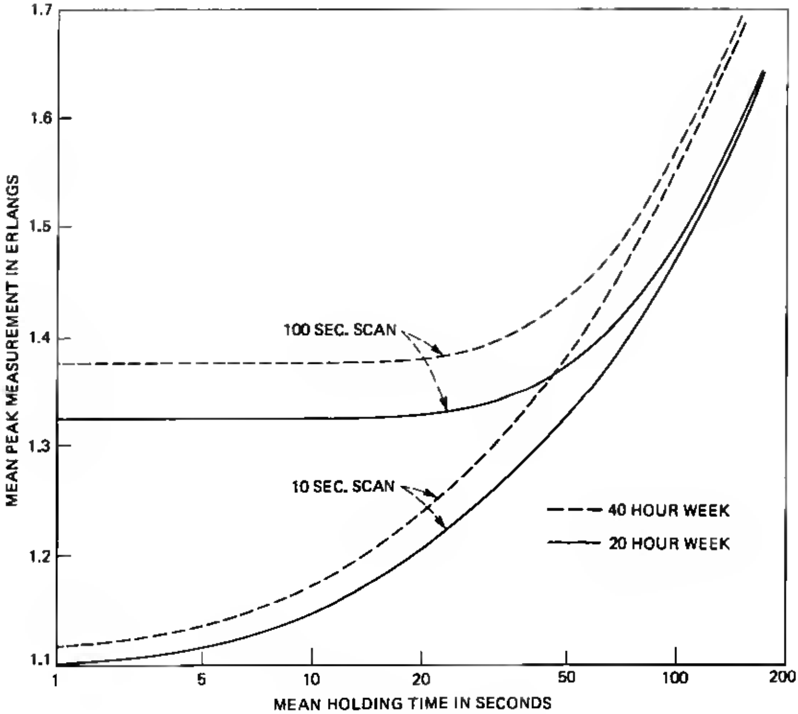

Fig. 5—Weekly peak hour measurements (1 erlang of source load).

the largest hourly measurement one does not necessarily find the hour of peak traffic, but one may find an hour when the scanning error is unusually large. For the model used in this analysis, this upward bias may be as much as 10 or 20 percent. It has also been shown that this bias is only significant in situations where the holding time is much shorter than the scanning interval. For those systems where the holding time exceeds the scanning interval, the bias introduced by the scanning process is negligible.

IV. ACKNOWLEDGMENT

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APPENDIX

Derivation of Mean Bouncing Busy-Hour Measurements

The following calculations extend the work of W. S. Hayward, Jr.[2] on the determination of switch count errors. Here, the offered traffic is assumed to have the following characteristics:

(*i*) Calls originate according to a Poisson process.

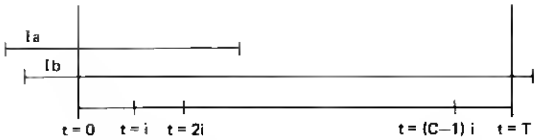

Fig. 6—Graphical interpretation of type I calls.

(*ii*) Holding times are exponentially distributed.

(*iii*) Incoming calls are processed immediately upon arrival.

(*iv*) Usage measurements for adjacent observation intervals are statistically independent.

Let i = the time between successive scans

τ = the mean holding time

a = the source load in erlangs

T = the length of the observation interval

$\beta = i/\tau$ = the number of holding times per scanning interval

$C = T/i$ = number of scans per observation interval

N = the number of observation intervals per day.

The observation interval begins with the first scan and is assumed to end i units after the last scan, and data are acquired by a sampling scheme. The contribution to the total usage of an observation period by one call is first determined and then modified to account for n calls where n is random with a Poisson distribution. Once the distribution of usage for one observation interval is known, the distribution for the maximum of N such observation periods is computed. The mean of this distribution yields the expected bouncing busy-hour measurement. In this analysis, two types of calls must be considered, those originating outside the observation interval, type I, and those originating within, type II. These may be further subdivided into class a for those ending within the observation period, and class b for those terminating outside the interval (Figs. 6 and 7).

Since calls have exponentially distributed holding times, the probability density for the length of a call beyond $t = 0$ is given by $p(t) =$

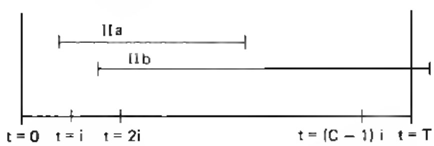

Fig. 7—Graphical interpretation of type II calls.

$(1/\tau)e^{-t/\tau}$. A contribution of K switch counts $(0 < K < C)$ is made by an arriving call if it terminates in the interval $[(K-1)i, Ki)$, and the contribution is C switch counts if the call exceeds $(C-1)i$ time units in length. The probability of measuring K switch counts, $\bar{P}(K)$, is given by

$$\bar{P}(K) = e^{-[(K-1)i/\tau]} - e^{-(Ki/\tau)} \qquad K = 1, 2, \cdots, C-1 \tag{11}$$

$$\bar{P}(C) = e^{-[(C-1)i/\tau]}. \tag{12}$$

A type IIa call makes no contribution of usage if it arrives in an interval $[ni, (n+1)i)$ $n = 0, 1, \cdots, C-1$ and terminates before the end of the same interval. The probability of zero switch counts becomes

$$P(0) = \sum_{n=0}^{C-1} \int_{ni}^{(n+1)i} \frac{dx}{T} (1 - e^{-[(n+1)i-x]/\tau}). \tag{13}$$

Similarly, a contribution of K switch counts by a IIa call is made if it originates in the interval $(ni, (n+1)i]$ and ends in the interval $[(n+K)i, (n+K+1)i)$ $n = 0, 1, \cdots, C-K-1$. The probability of this occurring is given by

$$P(K) = \sum_{n=0}^{C-K-1} \int_{ni}^{(n+1)i} \frac{dx}{T} (e^{-[(n+K)i-x]/\tau} - e^{-[(n+K+1)i-x]/\tau}) \tag{14}$$

$$K = 1, 2, \cdots, C-1.$$

With a type IIb call, K switch counts are measured when the call originates in the interval $((C-K-1)i, (C-K)i]$ and extends beyond $t = T$. The probability of a IIb call contributing K switch counts is

$$P(K) = \int_{(C-K-1)i}^{(C-K)i} \frac{dx}{T} e^{-(Ci-x)/\tau} \qquad K = 0, 1, \cdots, C-1. \tag{15}$$

Simplifying, the above expressions become:
Type I calls:

$$\bar{P}(K) = (e^{\beta} - 1)e^{-K\beta} \qquad K = 1, 2, \cdots, C-1 \tag{16}$$

$$\bar{P}(C) = e^{-(C-1)\beta} \tag{17}$$

Type II calls:

$$P(0) = 1 - \left(\frac{C-1}{C\beta}\right)(1 - e^{-\beta}) \tag{18}$$

$$P(K) = \left\{\left(\frac{C-K}{C\beta}\right)(e^{\beta/2} - e^{-\beta/2})^2 + \left(\frac{1}{C\beta}\right)(1 - e^{-\beta})\right\}e^{-K\beta} \tag{19}$$

$$K = 1, 2, \cdots, C-1.$$

The number of type I calls seen in an observation interval is a

random variable having a Poisson distribution with mean a, while the number of type II calls is Poisson-distributed with mean aT/τ. Following the laws governing the computation of probability generating functions of random sums of random variables, the generating functions for type I calls, $\phi_I(Z)$, and type II calls, $\phi_{II}(Z)$, are:

$$\phi_I(Z) = \exp\left(a\left\{\left(\sum_{K=1}^{C} \bar{P}(K)Z^K\right) - 1\right\}\right), \tag{20}$$

$$\phi_{II}(Z) = \exp\left(\frac{aT}{\tau}\left\{\left(\sum_{K=0}^{C-1} P(K)Z^K\right) - 1\right\}\right). \tag{21}$$

The measurements resulting from the two different types of calls are statistically independent. The generating function for the measurement due to both types of calls is

$$\phi(Z) = \phi_I(Z)\cdot\phi_{II}(Z) \tag{22}$$

or

$$\phi(Z) = \exp\left(\sum_{K=0}^{C} q_K Z^K\right), \tag{23}$$

where

$$q_0 = -a\{(C-1)(1-e^{-\beta}) + 1\}, \tag{24}$$

$$q_K = a\{e^{\beta} - e^{-\beta} + (C-K)(e^{\beta/2} - e^{-\beta/2})^2\}e^{-K\beta}, \tag{25}$$

$$K = 1, 2, \cdots, C-1, \tag{26}$$

$$q_C = ae^{-(c-1)\beta}.$$

To extract the probability distribution for the measured usage of one observation period, $\phi(Z)$ must be expanded into a power series. The coefficient of Z^K then becomes the probability of measuring K switch counts. Noticing that

$$\phi'(Z) = \left(\sum_{K=1}^{C} Kq_K Z^{K-1}\right)\exp\left(\sum_{K=0}^{C} q_K Z^K\right), \tag{27}$$

it is clear that $\phi(Z)$ solves the differential equation

$$\phi' - \left(\sum_{K=1}^{C} Kq_K Z^{K-1}\right)\phi = 0, \qquad \phi(0) = e^{q_0}. \tag{28}$$

Solving by infinite series techniques, a solution of the form $\sum_{n=0}^{\infty} b_n Z^n$ is obtained with

$$b_0 = e^{q_0}, \tag{29}$$

$$b_n = \begin{cases} \dfrac{1}{n}\displaystyle\sum_{m=1}^{n} mq_m b_{n-m} & n \le C \\ \dfrac{1}{n}\displaystyle\sum_{m=1}^{C} mq_m b_{n-m} & n > C. \end{cases} \tag{30}$$

Thus b_n is the probability of measuring n switch counts in an interval of length T with a source load of a erlangs. As each day consists of N intervals, with the measurement from each having the distribution $\{b_n\}$, P_n, the probability of obtaining a daily maximum of n switch counts becomes

$$P_0 = b_0^N, \tag{31}$$

$$P_n = \left(\sum_{K=0}^{n} b_K\right)^N - \left(\sum_{K=0}^{n-1} b_K\right)^N \tag{32}$$

$$n = 1, 2, 3, \cdots.$$

(The maximum has a value of n if and only if all observations yield measurements less than or equal to n, but it is not true that all yield measurements less than or equal to $n - 1$.) Since there are $T \cdot C$ switch counts per erlang, the mean bouncing busy-hour measurement is computed as

$$E(\text{BBH})_{\text{Sampled}} = \frac{1}{T \cdot C} \sum_{n=1}^{\infty} nP_n. \tag{33}$$

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